

Linear regression and least squares

1. Find the linear combination of the vectors $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ that best approximates the vector $\begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$. Is

this approximation exact? If you tried to solve $\begin{pmatrix} 2 & 1 & \vdots & 1 \\ 1 & -3 & \vdots & 7 \\ -2 & 1 & \vdots & -5 \end{pmatrix}$, are there no solutions, one solution or infinitely many solutions?

2. Find the linear combination of the vectors $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ that best approximates the vector $\begin{pmatrix} 2 \\ 8 \\ -6 \end{pmatrix}$. If

you tried to solve $\begin{pmatrix} 2 & 1 & \vdots & 2 \\ 1 & -3 & \vdots & 8 \\ -2 & 1 & \vdots & -6 \end{pmatrix}$, are there no solutions, one solution or infinitely many solutions?

3. Find the linear combination of the vectors $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ that best approximates the vector $\begin{pmatrix} 5 \\ 4 \\ 7 \end{pmatrix}$. Can

you explain the result?

4. Find the least-squares best-fitting linear polynomial in the variable t and least-squares best-fitting quadratic polynomial in the variable t that pass through the five points $(0, 1)$, $(1, 1)$, $(2, 1.2)$, $(3, 1.5)$, $(4, 2.5)$. Plot the five points and the two polynomials, and evaluate the polynomials at $t = 4$. You can use a program to help you find these polynomials, but you should know how to set up the system of linear equations you must solve.

5. Suppose the readings in Question 4 were how far north you were of given point based on GPS readings where the t value is 0 s, 1 s, ..., 4 s and there is an error with each of the readings. If you knew you were actually going at a constant speed, what is the best approximation of how far north you actually are at time $t = 4$ s? If you knew that you were approximately constantly accelerating during this time interval from 0 s to 4 s, what is the best approximation of how far north you are at time $t = 4$ s?